Math 1A, Sample questions for the final

Here are a few typical questions covering Chapter 5 and onward. For a more complete view, look at the homework! For questions on the earlier parts of the course, look at the midterms and sample midterms.

1. State carefully:
   (a) The fundamental theorem, part 1
   (b) The fundamental theorem, part 2
2. Prove directly from the definition (using limits of Riemann sums) that $\int_0^1 4x dx = 2$. (Recall that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. You need not memorize such summation formulas; I'll give them to you on the test.)
3. Water flows into a tank, the inflow rate at time t hours (after some reference time) being $r(t) = te^{-t^2}$ cubic meters per hour. How much water flows into the tank between times $t = 1$ and $t = 2$?
4. Find the area of the region bounded by the curves $y = e^{-x}, y = e^{2x}, x = -2$, and $x = 2$.
5. Find the volume common to two spheres, each with radius r, if the center of each sphere lies on the surface of the other.
6. The region bounded by the curves $y = x^3 + x^2$, $x = 2$, and the x-axis is rotated about the line $x = -1$. What is the volume of the resulting solid?
7. Compute $\int_0^{2/3} \frac{1}{4 + 9x^2} dx$.
8. The base of a solid is the triangular region with vertices $(0, 0), (3, 0)$, and $(0, 2)$. Its cross-sections perpendicular to the y-axis are semicircles. What is its volume?